AJNR

Volume Averaging Limitations of Computed Tomography

D. Goodenough, K. Weaver, D. Davis and S. LaFalce

AJNR Am J Neuroradiol 1981, 2 (6) 585-588
http://www.ajnr.org/content/2/6/585

Volume Averaging Limitations of Computed Tomography

D. Goodenough[1]
K. Weaver
D. Davis
S. LaFalce

A simple volume averaging model was shown to be inadequate for predicting effective computed tomographic attenuation values of mixtures of bone or air with soft tissue. Computed tomographic values derived from scanning stratified sub-slice thickness objects were shown theoretically and experimentally to have a non-linear dependency on relative fractional content and a surprisingly large dependency on spatial extent of the stratified substances. The mode was applied to the problem of computed tomographic values in thin, flat structures such as pools of blood in the subarachnoid space. The results show only small deviations from simple volume averaging theory for layers of low contrast substances such as blood and soft tissue, but potentially large deviations for layers of substances with high contrast differences such as bone, air, and tissue. This phenomenon explains certain artifacts and demonstrates rather fundamental problems in the accuracy of analytic reconstruction techniques. It may justify postprocessing correction or iterative approaches.

The problem of "volume averaging" in computed tomography (CT) was recognized early in its development [1–3]. It was noted that blood in the subarachnoid space and blood stained pia-arachnoid are generally distributed along thin and flat spaces, and their detection may be influenced strongly by partial volume phenomena [4, 5]. In our report, we show that volume averaging is a complex phenomenon involving not only the expected relative fractions of substances mixed in the slice, but also their spatial distribution. Thus, for a conventional axial scan, one needs to consider not only the relative proportions of the substances in the slice (Z) direction, but also the axial extent of the mixing. An important question is whether this spatial dependence phenomenon might influence the detection of thin, flat distributions of blood, such as subarachnoid hemorrhages.

A recent article by Thomas et al. [6] sought to justify the application of "simple volume averaging." We show that the justification for simple volume averaging can often be misleading and erroneous. The errors stem from limiting the analysis to sub-voxel dimensions (e.g., an independent voxel model as developed in the Appendix to the Thomas et al. article). Usually, these errors will not be significant in a case such as was considered by Thomas et al. of averaging spherical-type tumors in tissuelike surroundings. However, errors can become significant in air or bone averaging, particularly when thin, flat structures are involved.

The problem may be depicted as in figure 1, which shows a radially symmetric CT slice partitioned into two different homogeneous layers characterized by linear x-ray attenuation, μ_1 and μ_2, respectively. The situation constitutes a stratified rather than a random mixture, as may be encountered in layers of blood in the subarachnoid space and cisterns [5].

The radiation source is assumed to be monoenergetic and to have an effectively uniform beam profile in the Z direction. It is straightforward to predict the effective CT attenuation coefficient by routine logarithmic transformation of relative x-ray

This article appears in the November/December 1981 *AJNR* and February 1982 *AJR*.

Received August 26, 1980; accepted after revision June 22, 1981.

Presented in part at the annual meeting of the Radiological Society of North America, Dallas, TX, November 1980.

[1]All authors: Department of Radiology, George Washington University Medical Center, Washington, DC 20037. Address reprint requests to D. Goodenough.

***AJNR* 2:585–588, November/December 1981**
0195-6108/81/0206-0585 $00.00

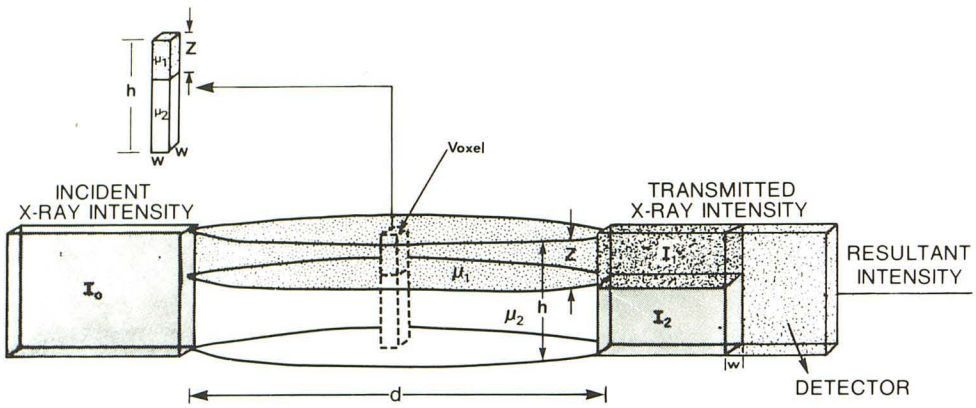

Fig. 1.—/Transmission of CT x-ray beam through object stratified into two homogeneous layers, μ_1 and μ_2; d is spatial extent of stratified layers. Z = thickness of μ_1; h = height of $\mu_1 + \mu_2$; w = width; I_0 = intensity of unattenuated beam; I_1 and I_2 = attenuated intensity at the exit of μ_1 and μ_2, respectively.

intensity values. Each of the partitioned regions will attenuate x-rays according to its respective μ value. The intensity of the unattenuated beam is denoted by I_0, and the attenuated intensity as I_1 and I_2 at the exit of the respective regions. The slice thickness is denoted by h, the fraction of μ_1 is Z/h, the fraction of μ_2 is $1 - Z/h$. The model could represent a thin sheet of substance μ_1 of thickness Z within a CT scan of surrounding substance μ_1 and slice thickness h.

Assume that the CT system establishes the attenuation properties of an object by determining x-ray transmission levels through known path lengths. The detector integrates the total acquired x-ray flux over its width w and height h. The total integrated detector signal with (D) and without (D_0) the object being present can then be determined by the following equations:

$$D = I_1 \cdot Z \cdot w + I_2 \cdot (h - Z) \cdot w \rightarrow \quad (1)$$

$$D_0 = I_0 \cdot w \cdot h \rightarrow \quad (2)$$

where

$$I_1 = I_0 e^{-\mu_1 d} \rightarrow \quad (3)$$

and

$$I_2 = I_0 e^{-\mu_2 d} \rightarrow \quad (4)$$

then

$$\mu_{eff} = \frac{1}{d} \log_e \frac{(D)}{(D_0)}$$

$$= -\frac{1}{d} \log_e \frac{[I_1 Z + I_2 (h - Z)]}{I_0 \cdot h} \rightarrow \quad (5)$$

$$= -\frac{1}{d} \log_e \frac{[e^{-\mu_1 d} \cdot Z + e^{-\mu_2 d} \cdot (h - Z)]}{h} \rightarrow \quad (6)$$

$$= \mu_2 - \frac{1}{d} \log_e \left\{ 1 + \frac{Z}{h} [e^{(\mu_2 - \mu_1) d} - 1] \right\} \rightarrow \quad (7)$$

The limiting values of the above equation yield the expected values in that for:

case 1, $Z = 0$, $\mu_{eff} = \mu_2$ ·

case 2, $Z = h$, $\mu_{eff} = \mu_1$ ·

case 3, $(\mu_2 - \mu_1)\, d \rightarrow 0$, $\mu_{eff} \rightarrow \mu_1 \frac{Z}{h} + \mu_2 \left(1 - \frac{Z}{h}\right)$ ·

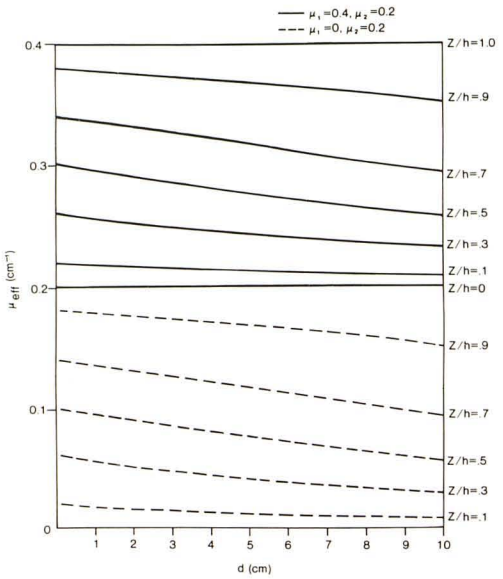

Fig. 2.—Predicted change in CT values as a function of spatial extent, d, and relative fraction Z/h of stratified layers of tissue (μ_2 = 0.2 cm^{-1}) and bone (μ_1 = 0.4 cm^{-1}) and of tissue (μ_2 = 0.2 cm^{-1}) and air (μ_1 = 0.0 cm^{-1}). The deviation from the simple volume averaging model, the ordinate values, is a function of increasing d. μ_{eff} = effective values for stratified mixtures.

Case 1 and 2 obviously correspond to the situation of no volume averaging. Case 3 corresponds to the often assumed first order equation for simple volume averaging. This linear equation will only be true when the product of the difference in μ values and the spatial extent (d) of the differing regions is small. The exact solution indicates a non-linear dependence not only on $(\mu_2 - \mu_1)$, *but also* on the spatial extent of the mixing (d). In the case of thin, flat layered distributions of blood after subarachnoid hemorrhages, d would represent the extent of the blood layer in the axial plane. If the equations are restricted to a single independent voxel as in the Thomas et al. [6] article, then the maximum value permissible for d is dictated by the voxel dimensions—the pixel width and slice width. In this restrictive case, where the averaging is developed in a sagital or coronal fashion, d will be no greater than the pixel width, a millimeter or so. Thus, the product of $(\mu_2 - \mu_1)d$ will be artificially restricted by the dimensions of the voxel. When the theory is developed to

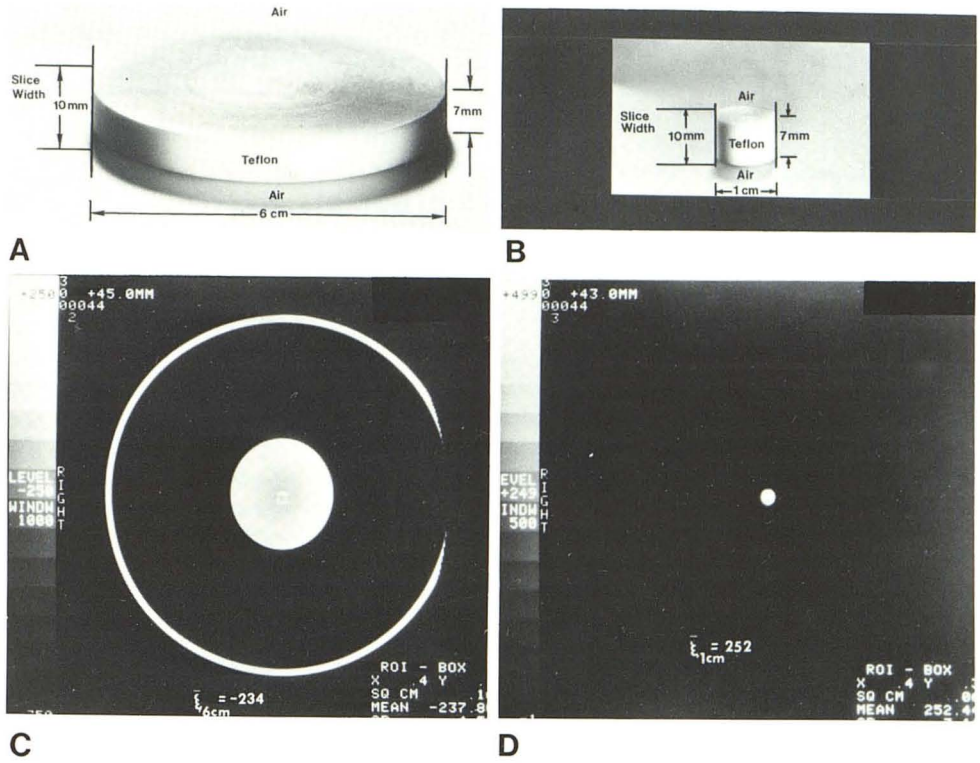

Fig. 3.—Results of an experiment scanning 7 mm thick Teflon disks of 6 and 1 cm diameters in air (**A** and **B**, respectively) with a GE 8800 scanner at 120 kVp and with a nominal 10 mm slice. Theoretical change in central CT number is 435, as predicted by equation 7, whereas the experimental results (**C** and **D**) show central CT value changed 486 CT numbers (−236–+252) as diameter changes from 6 to 1 cm. Small differences from model can be anticipated according to exact sensitivity profile and polychromatic effects.

the actual spatial extent, perhaps many pixel widths, of mixing, the product $(\mu_2 - \mu_1)d$ can no longer be assumed negligible, case 3 does not hold, and serious errors may occur from the simple volume averaging model.

Equation 7 is of more than simple mathematical interest in that it points out a seemingly fundamental limitation to analytic noniterative backprojection reconstruction techniques. A single integrated detector reading cannot yield the correct composition of a mixed volume without first knowing the spatial extent (d) of the volume! Moreover, the *exact* solution is exact only for the prescribed two compartment stratified model, for a monoenergetic beam, and for a uniform x-ray beam intensity. Appropriate modification of this model for beam profile and spectral composition is not fundamentally difficult and will be discussed in future reports.

Figure 2 shows the use of equation 7 to predict the effective μ_{eff} values for stratified mixing of dense bone ($\mu_1 = 0.4$ cm $^{-1}$) and soft tissue ($\mu_2 = 0.2$ cm $^{-1}$) as a function of the spatial extent (d) of the volume averaging. Similar changes of μ_{eff} as a function of spatial extent from mixing tissue ($\mu_1 = 0.2$ cm^{-1}) and air ($\mu_2 = 0$ cm^{-1}), are also shown. Large deviations from the simple volume averaging model (the ordinate values) can be seen with increasing d. That is to say, for any particular fixed fractional composition (Z/h), the CT numbers get increasingly biased toward the lower attenuating substance as the spatial extent (d) of the mixed layers gets larger.

Equation 7 and figure 2 predict the small thin disks of thickness less than the slice thickness should have CT numbers dependent on their diameter. This prediction was tested by scanning thin Teflon discs of various diameters. The disks were placed in air at the center of the scan volume (X, Y, and Z) of a GE 8800 scanner. For the case of these thin, small disks, differences in CT value due to differences in beam hardening should be minimal because of the small changes in spectral quality and small changes in net detector signal.

The excellent agreement between theory from equation 7 and the experiment is shown in figure 3. The changes predicted and empirical CT numbers as a function of radius are much larger than the statistical variations of a few CT numbers. Simply by changing the diameter of the thin disks from 6 to 1 cm, the predicted and experimental results show that the central CT number has changed from −234 to +252. The simple volume averaging model would have predicted no significant change in CT number with disk diameter!

Also, one can note the interesting cupping of CT values in the larger disk. This theoretically predicted phenomenon is found experimentally and results from the variable path lengths of Teflon presented to the beam as it crosses the disk, ranging from the disk diameter in the center of the disk to zero at the outside edge of the disk. The phenomenon should not be confused with spatial resolution overshoot effects, which usually result in a cappinglike phenomenon.

Moreover, equation 7 predicts that thin, sub–slice thickness, stratified objects that do not have constant path length in all possible sampling directions should have inconsistent effective attenuation values. Such predictions may be tested rather simply by cutting several pieces of aluminum with the same net area but different shape. Figure 4 shows the increasing differences in μ_{eff} as circular symmetry deviates toward a one-dimensional (asymmetric) area. Again, in contrast to the simple volume averaging model, dramatic differences in CT number result from the spatial distribution of

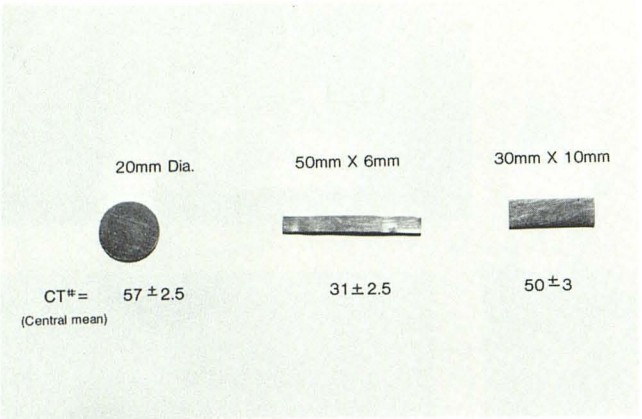

Fig. 4.—CT values resulting from 0.5 mm thick aluminum disks of differing shapes but equivalent areas on an EMI 5005 scanner at 140 kVp. The objects were situated in a central plane in a water bath. CT values changed from 31 to 57.

the volume averaged object.

We have shown that spatial extent is significant for strata or layers involving high contrast attenuation differences. We can now return to the questions posed in our introduction, whether the CT measurement of thin, flat pools of blood might also be influenced by the spatial extent phenomenon. Equation 7 was used to predict the change in CT values with increasing *d* for a thin flat, layer of a bloodlike material (nylon) and a tissuelike material (water). The results showed only a fairly small decrease in expected CT difference as the thin, flat blood pool increased in axial dimension (Table 1).

The spatial effect phenomenon will tend also to produce artifacts and is probably related to streaks seen in cranial transverse CT reconstructions at the level of the petrous bones. Independent analysis of this problem by Glover and Pelc [7] seems to confirm the relation of this artifact to volume averaging. Indeed dramatic improvements can be noted from using thin slices in the petrous region [7].

The final important observation to be drawn from equation 7 is that CT *sensitivity* profiles based on phantoms containing an aluminum ramp are subject to large theoretical errors, particularly as the width (spatial extent) of the ramp becomes large [8]. This error can be traced to the variable spatial extent of the averaged layers of aluminum and surrounding material that the ramp presents to the detectors, according to the orientation of the ramp and the sampling geometry under consideration. This error will be discussed in future papers.

To conclude, it has been shown theoretically and experimentally that the simple volume averaging model leads to significant errors in the CT attenuation values for mixtures of high or low density materials such as bone or air with tissue when they are distributed in stratified layers or thin, flat structures. On the other hand, spatial distribution will not significantly affect the detection of thin, flat blood pools beyond the normally expected reductions in contrast due to simple volume averaging. Because of the nature of artifacts propagated from higher contrast (bone and air), nonlinear, volume averaging artifacts, the general accuracy of CT numbers within the skull may be compromised.

TABLE 1: **Predicted CT Numbers from Volume Averaging Thin, Flat Structures of Various Diameters of Nylon (90) in Water (0) for 10 mm Slice Thickness**

Nylon Diameter (cm)	Slice Thickness (mm)	
	Nylon (1), Water (9)	Nylon (5), Water (5)
1	9	45
2	9	45
3	9	44
4	9	44
5	9	44
6	9	44
7	9	44
8	8	43
9	8	43
10	8	43

REFERENCES

1. Goodenough DJ, Weaver KE, Davis DO. Physical measurements of the EMI imaging system. In: Ter-Pogossian MM, Phelps ME, Brownell GL, Cox JR Jr, Davis DO, Evens RG, eds. *Reconstruction tomography in diagnostic radiology and nuclear medicine*. Baltimore, MD: University Park, **1977** : 223–243
2. Goodenough DJ, Weaver KE, Davis DO. Potential artifacts associated with the scanning pattern of the EMI scanner. *Radiology* **1975**;117 : 615–619
3. Zatz LM. The EMI scanner: collimator design, polychromatic artifacts, and selective material imaging. In: Ter-Pogossian MM, Phelps ME, Brownell GL, Cox JR Jr, Davis DO, Evens RG, eds. *Reconstruction tomography in diagnostic radiology and nuclear medicine*. Baltimore, MD: University Park, **1977** : 245–266
4. Lim ST, Sage DJ. Detection of subarachnoid blood clot and other thin, flat structures by computed tomography. *Radiology* **1977**;123 : 79–84
5. New PFJ, Scott WR, Schnur JA, et al. Computerized axial tomography with the EMI scanner. *Radiology* **1974**;110 : 109–123
6. Thomas SR, McLennan JE, Kereiakes JG, Neff R, Chambers AA, Lukin RR. Intracranial blood clot volume and geometric parameters determined from CT images. *Radiology* **1979**;133 : 741–746
7. Glover GH, Pelc NJ. Nonlinear partial volume artifacts in x-ray computed tomography. *Med Phys* **1980**;7 : 238–248
8. Brooks RA, Di Chiro G. Slice geometry in computer assisted tomography. *J Comput Assist Tomogr* **1977**;1 : 191–199