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## Flow dynamics for radiologists. I. Basic principles of fluid flow.

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### Flow Dynamics for Radiologists

#### I. Basic Principles of Fluid Flow

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Biorheology, the study of physiologic fluid flow in live organisms, is a rapidly expanding science. Hemobiorheology more specifically studies blood flow in normal and diseased vessels (1–10). Despite our intimate and daily association with flowing blood, angiographers generally have paid little attention to fundamental fluid dynamics and instead have concentrated our efforts on the anatomic depiction of disease through gray-scale images.

In fact, fluid flow is one of the basic foundations of all life. It also is fundamental to neurointerventional technique and treatments. Unfortunately, in living systems, fluid flow is difficult to study and model. When one attempts direct visualization of flow in either a live patient or an experimental animal, that flowing fluid all too soon changes or stops. Experimental problems are compounded in the head and neck, where vessels have focal dilatations, have many teleologically unexplainable curves, are visually opaque, and lie for much of their course within dense bone.

Most basic knowledge about flow and fluid comes from the engineering sciences, from studies that have used rigid, nontapering tubes, ideal (newtonian) fluids, and constant flows (11–19). In the head and neck, we have exactly the opposite conditions: (a) the arteries are filled with nonnewtonian fluid (blood becomes less viscous as it increases in velocity or as it is subjected to more shearing) (20–24); (b) the tubes are viscoelastic rather than rigid (8, 25, 26); and (c) the fluid does not flow at a constant rate but rather pulsates with a complex waveform. Complicating matters even more, the waveforms are different in various parts of the cardiovascular system (1–3, 27).

Although radiologists frequently believe that current imaging techniques give information about flow, in actuality our techniques almost

never do. Angiography provides elegant and precise anatomic detail but tells us nothing about the quantity of flowing blood. Doppler ultrasonography gives reliable data about velocity waveforms (28–31), but to date its flow information often is not reproducible. Magnetic resonance seems to offer real promise, and techniques are now being studied that should allow a noninvasive means of quantitating global flow in a specific artery not only overall, but also individual intra-arterial slipstreams (32–38). Because those individual slipstreams seem to be important in the production of atherosclerosis and berry aneurysms (8, 39), being able to visualize them may give us a noninvasive way to predict patients at risk.

We can build a foundation for understanding this complex science by considering three questions: What is fluid? What functions do fluids perform (why do they exist)? What happens when fluids flow?

#### What is Fluid?

Most simply, fluids are materials that pour and flow (Fig 1). They are indifferent to their container's shape. Almost anything that is not a solid is a fluid; that is, either a gas or a liquid.<sup>a</sup> In our world, the most common fluids are air and water.

Most classical rheologic studies have been carried out with Newtonian, waterlike fluids. When a fluid has no memory for a prior event and its viscosity does not change when a stress is applied to it (for example, when it is made to flow), it is considered Newtonian. Blood is not Newtonian; it becomes less viscous or thinner when it is sheared, as for example when it is made to pass through a capillary. This property is called *thixotropy*. The most common everyday thixotropic

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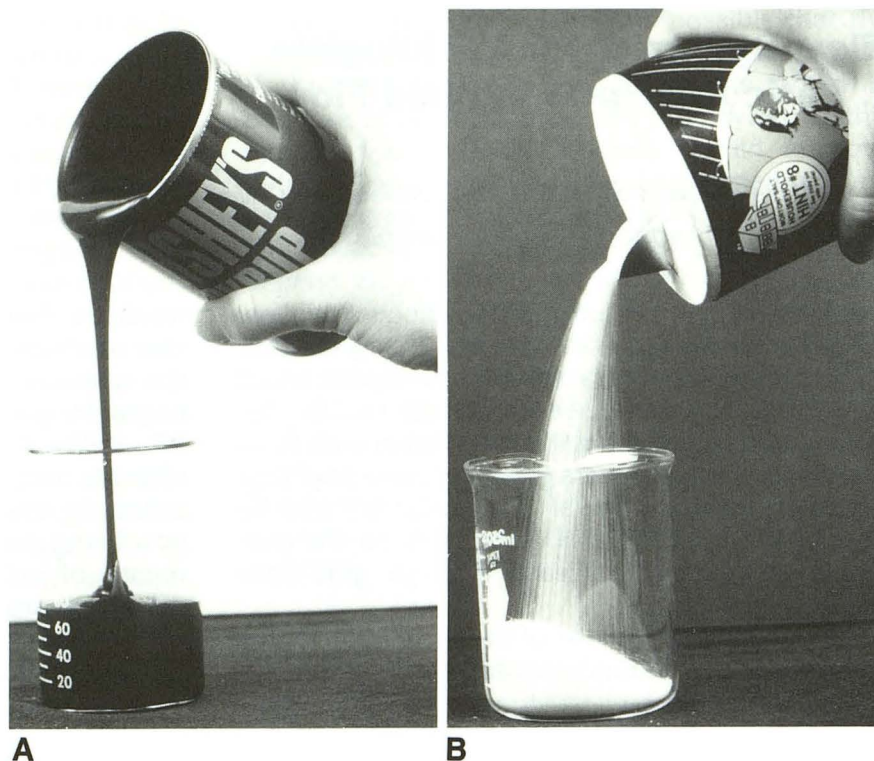
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Fig. 1. Pouring fluids and solids. A fluid is most easily defined as what it is not: anything that is not solid. Fluids pour, conforming to the shape of the container that holds them. The most common fluids are water, a liquid, and air, a gas. Fluids are continua and when moved from one container to another remain as much as possible a cohesive unit.

A, This liquid stream moves faster the farther it falls, because gravity accelerates it. If the fluid is to stay together (to be continuous), its stream diameter must narrow. This is an apparently simple but powerful concept.

B, Pouring divided or powdered solids (salt, for example) shows the difference. The salt particles are not continua. The crystals do not attempt to conform to the container, nor do they stay together.



liquid is ketchup.<sup>b</sup> Conversely, liquids that become more viscous when sheared are termed *rheopexic*. Some complex fluids are both viscous and elastic. Silly Putty (40) and blood are examples (Fig 2).

Fluids have mass. As a result, they have inertia and will resist motion. When set into motion, the system will then have energy, and the fluid will be able to do work. Whether that work is useful or damaging to the organism depends on the organism's ability to channel it effectively.

Finally, fluids have a peculiar and useful property called *viscosity*. Viscosity is simply each individual particle's attraction to its neighbor and particles' resistance to shearing stresses, to being pushed past each other. We know from experience that molasses is more viscous than water. It takes more energy to make it flow than water. We could supply that energy by a stronger pump, or we could put energy into the molasses in the form of heat and thus lower its viscosity. One way or another, more energy is required.

### What Functions Do Fluids Perform?

In most situations, a fluid's value lies in its ability to be an efficient method of transport. Whether it floats a coal barge from one river town to another, whether water or freon is heating or

cooling your home, or whether blood is transporting oxygen and metabolites, fluids are a most efficient means of transportation. Although blood handles bulk transport of nutrients, metabolites, and wastes, it carries energy, too. It transports that energy in the form of heat, from our core to the skin, where it is radiated away.

### What Happens When Fluids Flow?

Flowing fluids are all about energy and forces and work. To develop our understanding further, we must ask how those forces are applied to the system, how the energy is transferred from one point to another, and what work gets done on the container (the arteries) during the transfer. If we consider the following eight characteristics of fluid flow, we will have a good foundation for understanding the phenomena we encounter in the human body.

#### Forces

Newton's first law (the law of inertia) states that a body will stay at rest unless some force is applied to it. So, for a fluid to flow, some outside force must be applied to set it in motion. We generally apply force to a fluid with a pump. Once it begins to flow it will continue to move until another force damps its movement. The

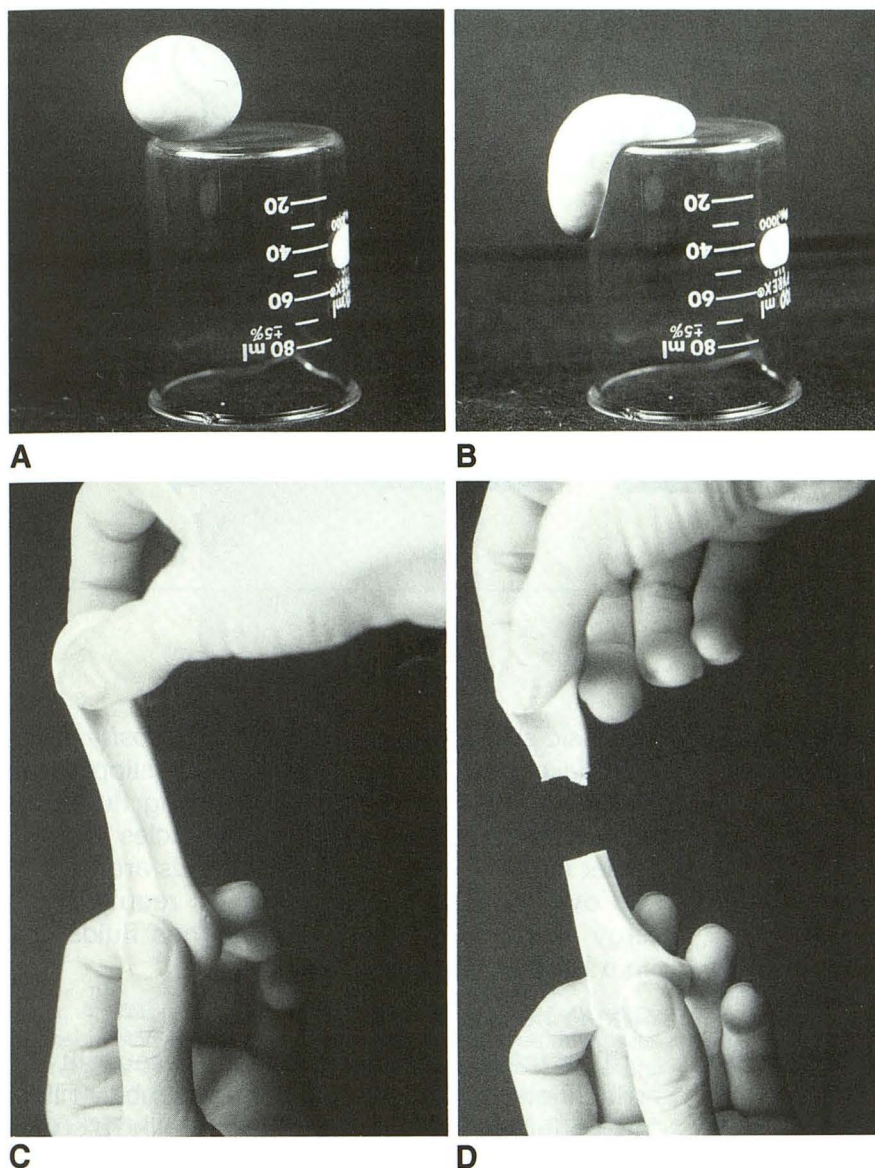


Fig. 2. Both Silly Putty (shown here) and blood are complex, non-Newtonian fluids. They do flow, but not in a laminar way. They have a property called viscosity, are elastic, and at the same time have a memory for prior events.

A, Being plastic, and having memory, the Silly Putty can be formed into a sphere.

B, Over time, and with force (here, gravity) the apparent solid flows, though slowly, because of its high viscosity.

C, Pulled (or sheared) slowly, it will change form (creep) and has some elasticity.

D, Pulled (or sheared) rapidly, its elastic limit is easily overcome, and it fractures.

force required to get it to move is expressed in Newton's second law: If we wish a certain acceleration, and if the fluid has a certain mass, simply multiplying the acceleration desired (or measured) by the mass of the fluid will give us the force required. Once that fluid is moving, it can do work that is useful or damaging, depending on factors to be considered later.<sup>c</sup>

#### *Energy Conserved*

Fluids flow in such a way as to conserve energy. Bernoulli found that if friction is ignored, when fluid enters a pipe with parallel walls, it will exit from the other end essentially unchanged in flow velocity, flow volume, or pressure. This concept is worth looking at in detail. Consider a

tube of certain diameter with fluid flowing through it at a constant rate (Fig 3). The fluid will exert some pressure on the wall. If we place a constriction in the pipe, the fluid velocity will increase as it travels through the constriction. If we measure the pressure at the constriction we will find the pressure decreased. This principle is used in a gasoline engine carburetor to suck liquid gasoline into a rapidly flowing air stream. The lower pressure in the constriction helps the gasoline vaporize.

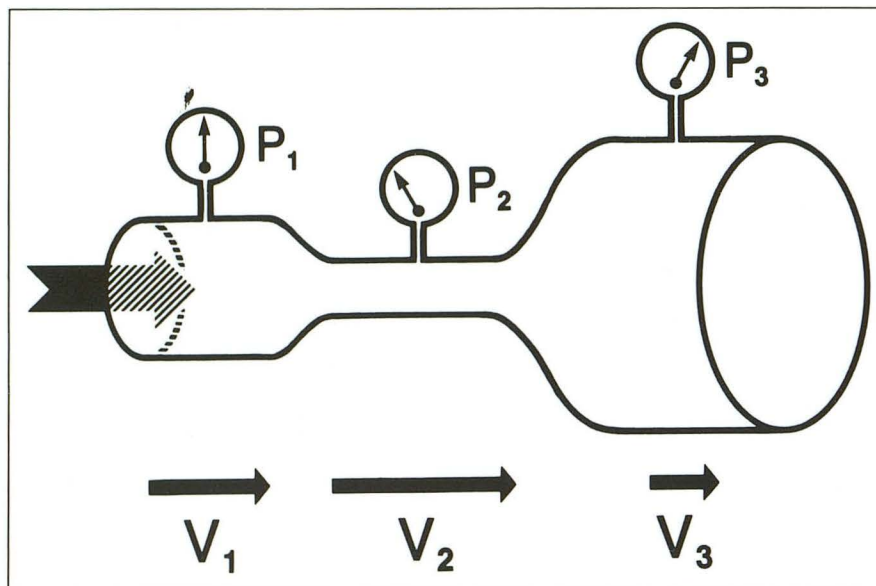
Let us now allow the same fluid to pass smoothly into a pipe larger than the original diameter. For the total fluid flow to remain constant, the velocity must decrease. What is not

Fig. 3. Bernoulli's principle. Assume that a fluid enters the system (here, a pipe) at a certain pressure and certain velocity. If we make a narrowing in the pipe, the velocity of the fluid will increase. As the velocity increases, the pressure decreases.

This narrowing has a common practical application. If a small tube leading from a gasoline tank is tapped into the narrowing, and the flowing fluid is air, the low pressure will suck gasoline into the air. Furthermore, the low pressure there will help vaporize the gasoline.

Let us continue our original pipe experiment by next expanding the tube so it is larger than the inlet pipe's size. The fluid then flows more slowly. What is not intuitively apparent is that the pressure in the pipe will increase beyond the baseline. The arrows below are actually vectors. Their lengths indicate relative velocity. Pressure changes are shown above the pipe in the gauges.

Remember, Bernoulli's principle ignores the effects of friction.



intuitively apparent is that simultaneously the pressure increases too. In summary, Bernoulli's theorem states that for perfect fluids, the total energy in the flowing system remains constant despite changes in velocity or pressure.

### Energy Lost

In the real world, no system is perfect; everything runs down. Mechanical energy is lost, usually to heat and noise.

Fluids tend to flow smoothly, but if we increase the fluid's velocity beyond a certain value for the system containing it (see the discussion of Reynolds numbers and viscosity, below), the smoothness of flow suddenly departs, there is random motion of fluid particles, heat is produced, and the energy needed to pump the fluid increases dramatically. When this happens, the flow is termed *turbulent*.

Second, in human arteries there may be some impediment in the fluid's pathway, such as an atherosclerotic plaque. The impediment may not result in true turbulence, but *drag* is produced. Drag is simply a resistance to flow caused by the impediment. The drag generates heat, and some mechanical energy is lost. Under certain conditions, the constriction may produce regular eddies. These become damped as they pass downstream. Known as *eddy shedding*, these disturbances usually produce a bruit (Fig 4).

Ultimately the explanation for these losses lies within the fluid itself; fluid particles have an

intrinsic stickiness, which we call *viscosity*. Newton described viscosity as a lack of slipperiness between adjacent layers of a moving fluid. It is this internal attraction of the particles to each other that must be overcome if fluids are to flow. To overcome viscosity, energy is required. Energy requirements are minimal when fluids flow in a laminar fashion.

### The Principle of Continuity

This principle bears repeating, because it is a critical one to understand and remember: Fluids are continua, not discrete entities. Unlike a crystal of salt that can be mechanically divided into smaller and smaller pieces, fluids tend to stay together whether they are contained by a pipe or vessel, whether they lie adjacent to a gas or a different fluid, or even when they are flowing within the same fluid. The most common example of continuity is water flowing from a faucet. The stream narrows as the water accelerates under the effect of gravity, but the water stream tends to stay together. The principle also explains how a stream of a liquid can pass through a body of the same liquid. A cold spring entering the bottom of a sun-warmed lake can be felt by a swimmer. Such an invisible stream holds together surprisingly well until dispersed by the currents and by the friction of the tube's edges against the stiller and warmer water. This powerful principle explains more subtle fluid behavior too, especially at bifurcations (Fig 5).

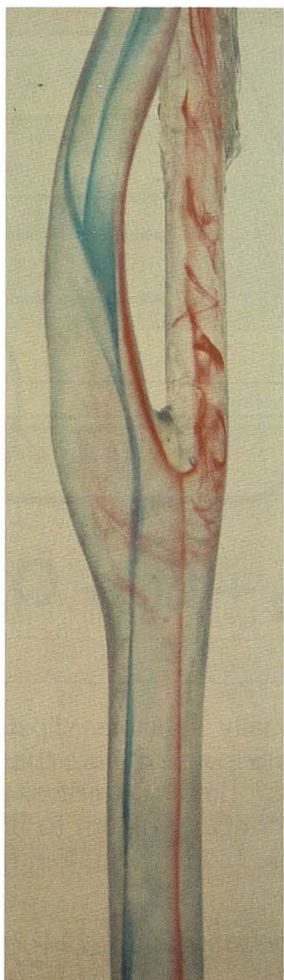


Fig. 4. This is an accurate elastic transparent model of a nearly normal human carotid bulb. The patient had a small atherosclerotic plaque at the carina. Regular eddies are produced as the fluid passes this plaque. The eddies are damped as the fluid flows downstream. Termed *eddy shedding*, this kind of disturbance usually produces an audible bruit. The slipstreams have been made visible by injecting isobaric colored dyes. The phase is peak systole.

### *Slipstreams (or Streampaths)*

A problem becomes immediately apparent when we consider what happens within the flowing stream itself. Unless there is some disturbance, the interactions within the stream are generally invisible. Our goal is to make the flow visible without disturbing it; we need to see the grain in the stone. Visualization is made possible by isobaric dyes, particles, or small bubbles (41–44) or, more elegantly, by using the coherent light from a laser and measuring the Doppler shift as the particles move (45, 46).

When there is no flow disturbance, when the tube walls are rigid, and when the fluid is Newtonian, the fluid flows as a series of sleeves

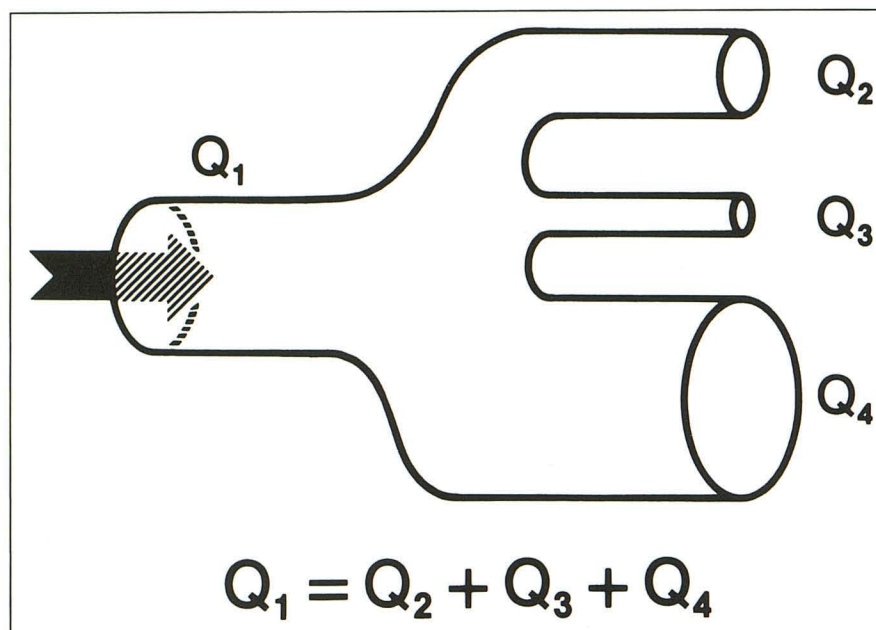
passing over each other. The fastest sleeves are in the center, the slowest away from the center near the wall. Disturb the flow momentarily or stop it; it will not matter. When the flow begins again, the motion of the sleeves resumes. The pathways of the fluid in sleeves have been termed *slipstreams* or *streampaths*.

By measuring the distance a particle flows during a short period we can measure velocity vectors for the various slipstreams. It is interesting (and elegant) that when all the vectors' leading edges are viewed from above, they describe a parabola (Fig 6). This parabolic profile does not develop instantaneously. Let us measure slipstream vectors as water enters a tube from a large holding tank (as in Fig 7). The slipstream parabolic profile develops over a certain distance. When fluid enters the tube, essentially all of the slipstream vectors are the same, and the flow is called *entry zone* flow or *plug* flow. Take an ultrasound transducer and move its Doppler probe across a common carotid artery. Sample all through the stream rather than just in the center. The peak velocities are essentially equal; there is no parabolic profile even though you are sampling 30 or 40 vessel diameters beyond the carotid origin. Entry zone flow is still present. We do not know why.

Let us now consider how slipstreams interact. Assume that fluid is moving in a tube as in Figure 6A. Notice that at the wall, there is no motion; in the center motion is rapid. Although particles cannot cross these slipstreams, and although there is no visible exchange of fluid from one lamina (layer) to the other, the particles do interact, transferring energy from one slipstream to the next. The slower outer layers tend to retard the faster central layers; the faster central layers tend to accelerate the slower. This is the effect of viscosity. It is possible to measure a *gradient* of velocity between successive laminae. The higher this so-called velocity gradient, the greater the energy transfer. The amount of energy transfer that takes place is termed the *coefficient of viscosity*. It best can be thought of as a measure of the amount of drag or impedance to flow that one slipstream of fluid exerts on the adjacent one, and is more practical for radiologists to understand conceptually than mathematically.<sup>d</sup>

Viscosity was formerly expressed in units of poise (after Poiseuille). As a useful reference, water has a viscosity of about 1 centipoise. Static human blood at 37°C is about 3.5 centipoise. Today a world-wide system of units, the *Système*

Fig. 5. The principle of continuity. Assume a certain flow enters the tube from the left. As it passes into the branches, it may slow down or speed up depending on the branch diameters, but the same volume exits as it enters. We have ignored the effects of gravity and viscosity in this example, but they are easy to factor in.  $Q$  indicates volume of flow per time.



International, has come into general use. The Système International is a metric-based system that allows easy conversions from one equation to another. Thus, we no longer use the term *centipoise* and instead express viscosity as newton seconds per square meter.<sup>e</sup>

**The No-Slip Region.** In a flowing stream, each fluid particle tends to flow by its neighbor, and because of the cohesive forces and attractions between them, each influences the other. As they approach the vessel wall, however, other forces come into play. At the wall, for all practical purposes, no fluid flow occurs. Regardless of whether the container is wetted or not (eg, whether it has a hydrophilic or hydrophobic surface), when fluid particles lie next to the walls of perfect systems, there is no downstream flowing motion. This is seen during angiography when hyperbaric contrast agent falls against a posterior vessel wall. Because carotid flow is not perfect or axis-parallel, eventually the contrast agent washes away. This region adjacent to the wall is called the *no-slip* or *no-flow* area. We can conceptualize the no-slip region (which may be an important factor in atherogenesis) as the region where a fluid behaves almost like solid.

**The Boundary Layer.** As slipstreams are observed even a small distance in from the wall, velocities increase rapidly. This region of rapid velocity increase, which can be characterized mathematically, is called the boundary layer (Fig 6D).

Though usually invisible, slipstreams are real and powerful. Slipstreams are the central core of the science of fluid dynamics. It is the fluid dynamicist's critical problem to find, to see, and to study these hidden slipstreams without disturbing them.

#### *Kinds of Slipstream Flows*

Industrial engineers use different kinds of flow depending on their needs. A chemical engineer wants to move gasoline from one portion of a cracking plant to another with no turbulence to keep pumping costs minimal. On the other hand, a paint chemist wants the maximum amount of turbulence to get the quickest pigment mixing through the paint solvent, thus reducing mixing costs. In the living organism conditions are more complex, but survival, in most situations, would favor the most efficient flows.

**Laminar Flow.** Laminar flow is flow at any velocity at which the slipstreams pass over each other smoothly and without disturbance. The leading edge of the velocity vectors traces a parabola. Laminar flow can be defined mathematically (see note d). Laminar flow is seen in efficient and well-run fluid dynamics laboratories and is rare elsewhere in the world.

**Turbulent Flow.** Increase the flow beyond a certain point, and the smoothness suddenly and violently departs. Slipstreams become ragged and disorganized and there is great energy loss. The best analogy is the pool at the base of a waterfall:

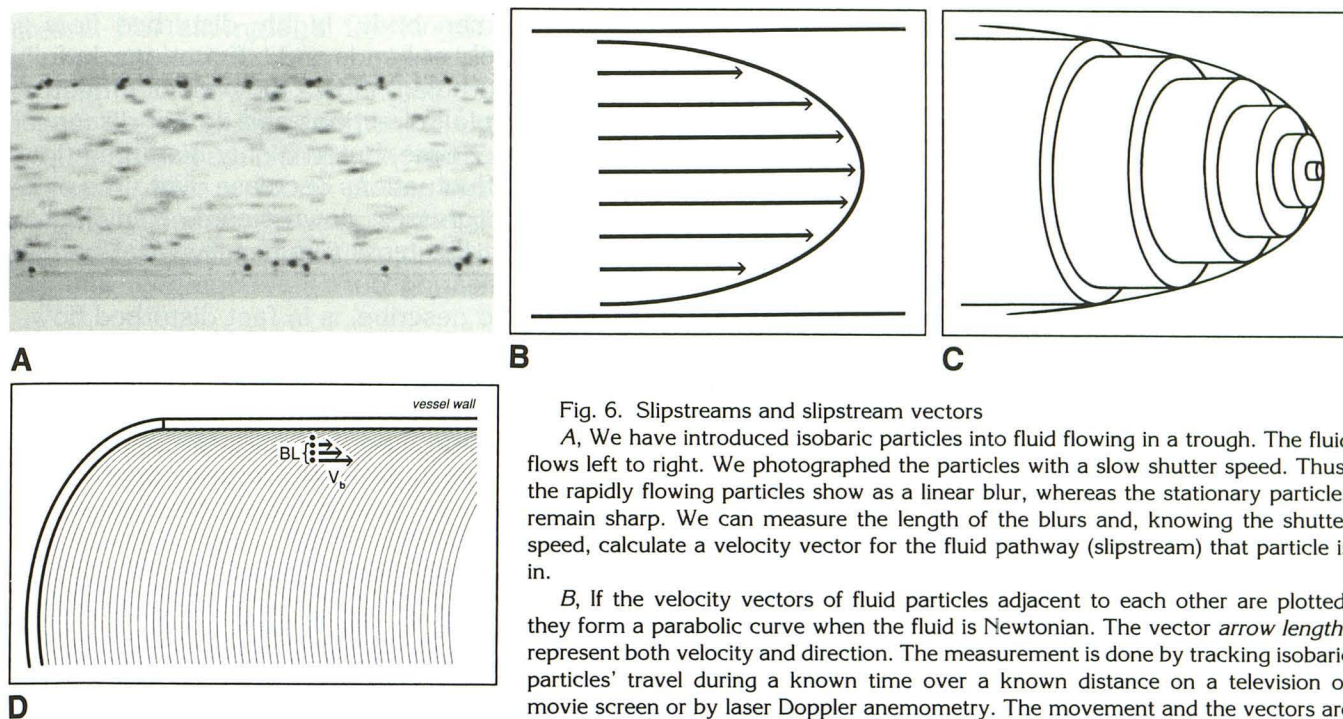


Fig. 6. Slipstreams and slipstream vectors

A, We have introduced isobaric particles into fluid flowing in a trough. The fluid flows left to right. We photographed the particles with a slow shutter speed. Thus, the rapidly flowing particles show as a linear blur, whereas the stationary particles remain sharp. We can measure the length of the blurs and, knowing the shutter speed, calculate a velocity vector for the fluid pathway (slipstream) that particle is in.

B, If the velocity vectors of fluid particles adjacent to each other are plotted, they form a parabolic curve when the fluid is Newtonian. The vector *arrow lengths* represent both velocity and direction. The measurement is done by tracking isobaric particles' travel during a known time over a known distance on a television or movie screen or by laser Doppler anemometry. The movement and the vectors are shown diagrammatically in two dimensions.

C, Remember though that arteries are three-dimensional. Flow is best conceptualized as a series of concentric thin sleeves, the central-most having the highest velocity. The vectors taken together create a *paraboloid*.

D, The no-slip area and boundary layer. Near the wall, there is no motion. Fluid acts almost as a solid in this region. As distance increases from the wall, slipstream velocities increase rapidly. For practical purposes, the thickness of the boundary layer is the distance from the wall to the velocity vector that is moving about 1% as fast as the fastest vector in the center of the stream.

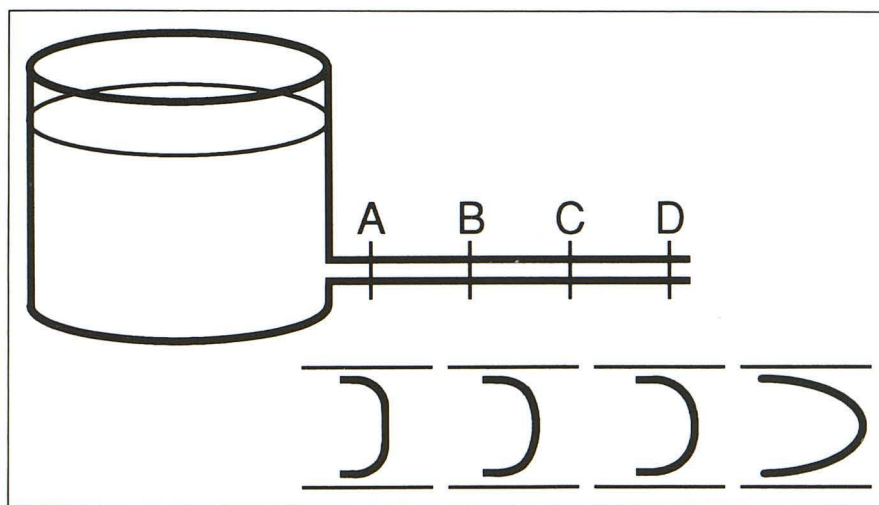


Fig. 7. Development of parabolic velocity profiles. A large volume of fluid is present in the tank at the *left*. A small pipe is led from the inferior portion of the trough, and fluid is allowed to flow from the tank into the tube. If one measures velocity vectors across the tube's diameter near the beginning of the tube, all of the velocity vectors are essentially the same. This is called *entry zone* flow or *plug* flow. As fluid progresses down the tube, it assumes the parabolic flow velocity pattern characteristic of Newtonian flow. It is interesting that when an ultrasound probe samples the carotid artery 20 to 30 diameters from the aorta (its entry zone) that entry zone flow is still seen.

There is scant forward flow, but massive random fluid motion.

Perfect laminar flow—and the other extreme, frank turbulence—are relatively rare. Most fluids behave otherwise. The swirls in a river, clouds passing overhead, and the mirage over a hot asphalt road are not quite smooth; neither are they frankly turbulent. In fact, the vast majority of flow in our world occurs in a transition region

that is neither smooth nor turbulent. How can we best understand this complexity?

This transitional region at which fluid flow changes from smooth, efficient flow into frank turbulence has been little studied, for it is difficult to characterize. However, it is critical to the life sciences, because many biologic processes and flow phenomena occur in this region. Hussain (47) and Liepsch (48) have defined types of flow

as *laminar* (fully axis-parallel), *nominally laminar*, *highly disturbed*, and *fully turbulent*.

We have come to use additional modifiers to characterize disturbed flows. If the disturbance of flow has a rhythm or periodicity to it, we use the term *harmonic* disturbed flow. An example is the eddies that are shed downstream from an atherosclerotic plaque (Fig 4). If the flow seems more chaotic, as in model aneurysms (49) we recommend the term *chaotic* disturbed flow (Fig 8). This indicates that the velocity fluctuations are irregular (50). This type of flow is extremely rare in the normal arterial system. Normally we observe a *periodic* flow created by the pulse wave. A precise description for this flow was given by Lieber (51).

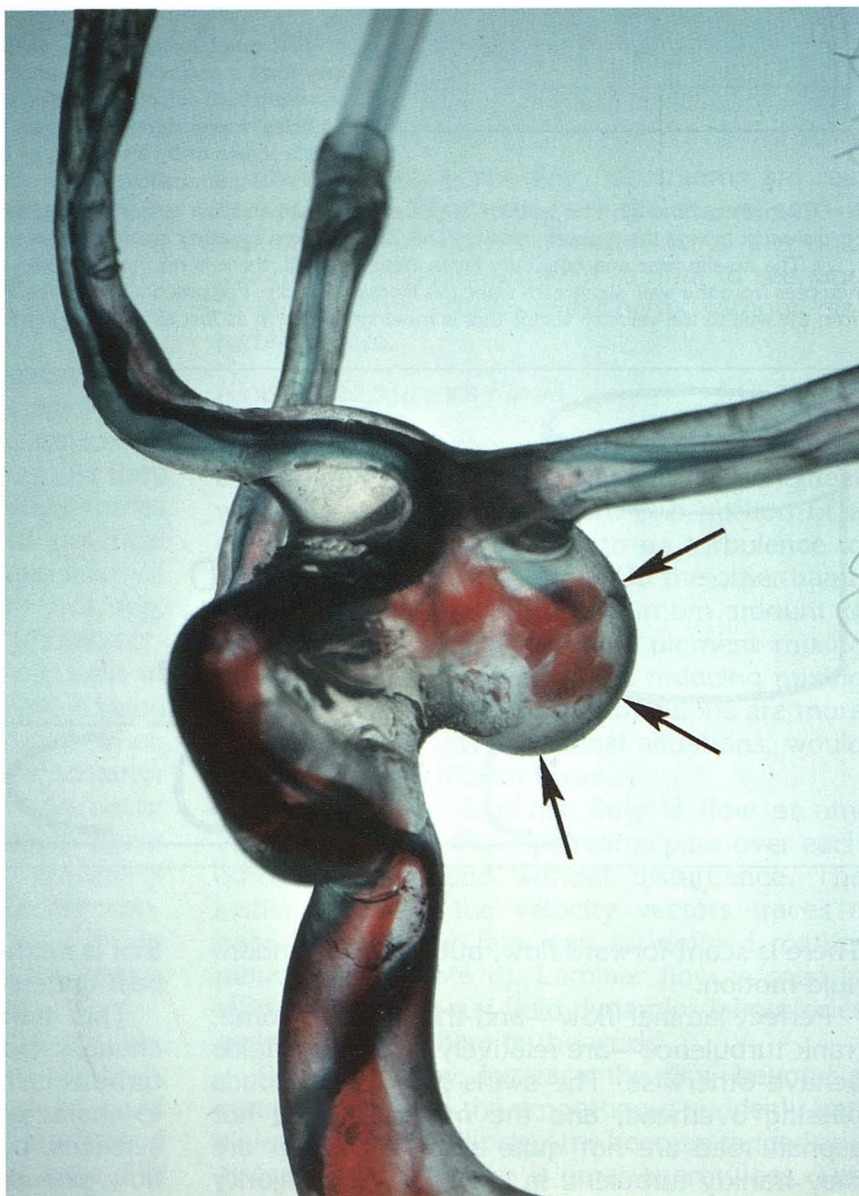
Fig. 8. Human carotid artery, transparent elastic model, slight left anterior oblique view. Arrows show a bulbous posterior communicating artery aneurysm. Isobaric dye injection. Note the random irregularity of the dye stream in the aneurysm. Frequently, we found no mixing (depending on the Re). Many dye streams left the aneurysm and passed *cohesively* and nonturbulently into a particular carotid branch, and the disturbed flow became laminar again.

This kind of flow is termed *highly disturbed*. The disturbance is probably best explained with chaos theory and seems to be rare in the human vascular system.

In the human body, highly disturbed flow is found normally at bends and bifurcations. In fully developed turbulent flow, the velocity fluctuations move in all directions and do not disappear locally or over time, but in highly disturbed flow, the velocity fluctuations decrease over time or as the fluid progresses downstream. Radiologists often misuse the term *turbulence*. What we are seeing and hearing during sonography, and are attempting to describe, is in fact disturbed flow.

#### *The Poiseuille Principle<sup>f</sup>*

Poiseuille studied the effects of pipe lengths and diameters on fluid flow (52). He found that if the length of a pipe were decreased, all other things remaining constant, fluid flow would in-



crease. For example, if the length of a pipe is decreased by 10%, we can expect a 10% increase in fluid flow. However, changing the diameter of a pipe causes a disproportionate change in flow. In quantitating the change, he found that diameter increase yields a fourth-power increase in flow volume. Thus if we double the diameter of a catheter, we will increase the quantity of contrast agent delivered 16-fold (ie,  $2^4$ ).

### *The Reynolds Number*

Reynolds (53) proposed a formula that allows comparison of the two fundamental opposing forces in flowing fluids: inertia and viscosity. His experiments showed a relationship between velocity, diameter, density, and viscosity. He found that multiplying the fluid's velocity by its density by the diameter of the vessel, and dividing by the fluid's viscosity gave a constant. This constant has been subsequently named the *Reynolds number*, abbreviated *Re*.<sup>9</sup> This important concept allows the rheologist both to manipulate study systems to optimize them and to understand real-time physical events better. For each particular system studied, the *Re* at which turbulence begins can be measured. For example, in a smooth, perfectly cylindrical rigid tube, an *Re* greater than 2300 leads to turbulence. Below an *Re* of 2300, flow in that system is laminar, even if there are some disturbances.

In our world, Reynolds numbers have a tremendous range—from one through 100 000 000 000 000 ( $10^{14}$ )—as shown by the following examples (from Vogel [19]).

|                                          |             |
|------------------------------------------|-------------|
| Gray whale swimming at 10 m/s            | 300 000 000 |
| Dragonfly flying at 7 m/s                | 30 000      |
| Red cell in carotid artery, about 6 cm/s | 200         |
| Sperm advancing the species at 0.2 mm/s  | 0.03        |

It is a common misconception that at one specific *Re*, all fluid flows will become turbulent. In reality, the *Re* is an average and is specific for a particular system.

Development of a familiarity with the ranges of *Re* is important for understanding biologic systems. In the center of the carotid artery, for instance, *Re* is about 200. However, this is a calculation based on *average* flow. It is occasionally useful to calculate an instantaneous or localized *Re* for a small sample of flowing fluid, for example, at the end of diastole and near the wall. This will become important as complex arterial flows are more fully studied (54–56). At an *Re* of 200, inertial forces predominate; at *Re* of 1 to 2 (ie, in the capillary bed, near the arterial wall, or

near some bifurcations), viscosity and viscous forces predominate.

### **Conclusion**

The actual study of fluid flow is complicated. Even more, it is messy. It is difficult for the experimenter to do a neat experiment with liquids, because tubings do come apart, usually at the most inopportune time, almost invariably damaging delicate measuring equipment and floors of the laboratory, and sometimes even inconveniencing the neighbors on the floors below. However, the visual appreciation of the results of those research efforts shows an elegant, complex, and fascinating dynamic. The images appeal aesthetically to all, but especially to radiologists, physicians who are predominantly visually oriented. Because we are primarily visually centered, we have placed the formulas as notes at the end of this article. We emphasize, however, that the basic principles are expressed best with mathematical formulas. Ultimate understanding and the proof of any new idea come by measuring, then by putting numbers on our observations.

Even the most casual critical analysis of each principle described in this introduction brings immediately to mind important everyday exceptions. All of these exceptions are useful in our lives and in our profession, but nonetheless, a thorough grounding and foundation in the basic principles is necessary so that we can advance our craft and understand novel observations in sonography, magnetic resonance angiography, and even angiography.

### **Acknowledgments**

The senior author thanks Dr Stephen Vogel for the inspiration to begin to wonder about flow in human cranial vessels. His book, *Life In Moving Fluids*, made me realize that fluid dynamics is a critically important part of the radiologic sciences, that we have generally ignored the study of blood flow despite our great experience with angiography, and more importantly, that a radiologist far removed from mathematics and the basic sciences should be willing to contribute observations to this discipline. If the reader has time for only one background text, it must be Vogels' *Life In Moving Fluids* (19).

I cannot thank my coauthor and friend, Dr Dieter Liepsch, enough. He has been unfailing in his support and infinitely patient when discussing the mathematical concepts that underlie the principles.

We also thank Ms Carol Gates and Ms Barbara Dean for the many manuscript revisions; Dr Stephen Hecht, Ms Cathy Fix, and Ms Joyce McLean for their outstanding

editorial assistance; and Ms Phyllis Stookey for the clear illustrations.

## Notes

<sup>a</sup> The fundamental differences between solids and liquids are explained by the forces and distances involved. A solid's basic building blocks (ie, the nuclear masses) are separated by a distance approximately the diameter of the nuclear mass and are held together tightly by powerful bonds. They are able to vibrate but can slide past each other only when a great shearing force is applied.

A fluid's building blocks or particles are separated the same order of distance, but the attractive forces between the particles are much weaker. These weaker attractive forces allow the particles to slide past each other.

A gas's particles are more widely separated than those of a liquid's but are still relatively close, usually several orders of magnitude of the particles' diameters. In gases, the attractive forces are even weaker than those attracting liquid's particles.

<sup>b</sup> As a child, you may have thought that the ketchup problem was your imagination or your impatience. Remember how you waited what seemed like forever for the ketchup to pour from the bottle? Then when it started, it came out too fast. Before you could turn the bottle upright, it completely covered your french fries. This was not your imagination or childlike lack of control; ketchup's viscosity lessens as it begins to flow. It is a thixotropic liquid.

<sup>c</sup> Newton's first law states that a particle continues at rest or in motion until or unless acted on by an external force. The second law states that when a force acts on a particle of mass, that particle will be accelerated: force = mass  $\times$  acceleration. This is useful in fluid dynamics because we can divide the forces into components: gravity force + pressure gradient force + viscous force = mass  $\times$  acceleration. The third law simply states that for every action there is an equal and opposite reaction. This law will become important when we consider the result of a pressure injection through the tip of a catheter.

<sup>d</sup> Call the difference between the two velocities  $du$ . Then if we compare this with the difference in radius of the two vectors ( $dr$ ), we get a function  $du/dr$ . This is the definition of a term called *shear rate*,  $\gamma$ . Thus  $\gamma = du/dr$ .

We can increase the stress on the fluid by making it flow faster. If we measure shear rates at various viscosities ( $\eta$ ), and plot the results, we get a straight line if the fluid is Newtonian (Fig 9)

<sup>e</sup> The Système International uses meters, kilograms, and seconds as its basic units, but is coherent throughout, and calculations in this system yield unit quantities. Within the system, no conversion factors are required.

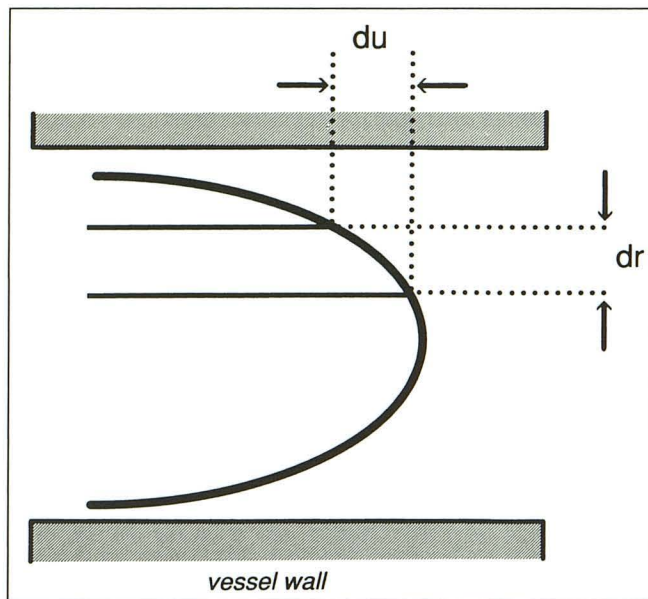


Fig. 9. If we measure velocities of a Newtonian fluid at two points in the tube, say at  $u$  and  $u_2$ , the velocity will be higher near the center.

For example, the unit of force is the newton (N).

$$1 \text{ N} = \frac{1 \text{ kg} \cdot 1 \text{ m}}{\text{s} \cdot \text{s}}$$

or

$$\text{N} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$$

and thus we can express pressure as  $\text{N}/\text{m}^2$ .

This avoids having to calculate the conversion factor for millimeters of mercury to some metric numbers. To get an idea of relative magnitudes, a mean arterial pressure of 100 mm Hg = 13 300  $\text{N} \cdot \text{m}^{-2}$ . Other useful conversion factors are: 1 cm  $\text{H}_2\text{O}$  = 98.1  $\text{N} \cdot \text{m}^{-2}$  (or about 100), and 1 poise = 0.1  $\text{N} \cdot \text{s} \cdot \text{m}^{-2}$ . In this system, dynamic viscosity is given in pascal seconds (Pa s) or, more conveniently, millipascal seconds (mPa s) and yields an expression of *force times area*.

<sup>f</sup> Poiseuille in 1840 proposed the following law:

$$\Delta P = \frac{Q \times \mu \times L \times 8}{\pi \times r^4},$$

where  $\Delta P$  is change in pressure;  $\mu$ , fluid viscosity (dynamic);  $L$ , pipe length;  $Q$ , flow volume;  $r$ , pipe radius; and  $\pi$ , 3.14.

This law was also proposed 1 year before by Hagen, and is sometimes called the Hagen-Poiseuille equation. It makes the important statement that flow is a fourth-power function of radius.

<sup>g</sup> The Re compares viscous to inertial forces in a fluid system and is dimensionless:

$$\text{Re} = \frac{\rho \times D \times U}{\mu},$$

where  $U$  is velocity of flow (free stream velocity);  $D$ , artery diameter;  $\mu$ , dynamic viscosity; and  $\rho$ , fluid density. The number is dimensionless; he found that when each factor was changed, it was just as quantitatively effective as changing another.

## References

- Bergel DH, ed. *Cardiovascular fluid dynamics*. London, New York: Academic Press, 1972
- Burton AC. *Physiology and biophysics of the circulation, an introductory text*. 2nd ed. Chicago: Year Book Medical, 1988
- Caro CG, Pedley TJ, Schroter RC, et al. *The mechanics of the circulation*. New York: Oxford University Press, 1978
- Leyton L. *Fluid behavior in biological systems*. Oxford, UK: Clarendon Press, 1975
- Lighthill J. *Physiological fluid mechanics*. New York: Springer-Verlag, 1972
- McDonald DA. *Blood flow in arteries*. 2nd ed. Baltimore: Williams & Wilkins, 1974
- Milnor WR. In: Mountcastle VB, ed. *Principals of hemodynamics in medical physiology*. 14th ed. St. Louis: Mosby, 1980:1017-1039
- Stehbens WE, ed. *Hemodynamics and the blood vessel wall*. Springfield, Ill: Thomas, 1979
- Stehbens WE, ed. *Pathology of the cerebral blood vessels*. St. Louis: Mosby, 1972
- Taylor MG. Hemodynamics. *Ann Rev Physiol* 1973;35:87-116
- Batchelor GK. *An introduction to fluid dynamics*. Cambridge, UK: University Press, 1967
- Bird RB, Stuart WE, Lightfoot N. *Transport phenomena*. New York: Wiley, 1960
- Byte FM. *Viscous fluid flow*. New York: McGraw-Hill, 1974
- Hertel H. *Structure form and movement*. New York: Reinhold, 1966
- National Bureau of Standards (United States). *The international system of units*. Washington, DC: US Government Printing Office, 1977
- Machtly EA. *The international system of units physical constants and conversion factors*. Washington, DC: National Aeronautics and Space Administration, 1973
- Streeter VL, Wiley EB. *Fluid mechanics*. 7th ed. New York: McGraw-Hill, 1975
- Segal BL, Kilpatrick DG. *Engineering in the practice of medicine*. Baltimore: Williams & Wilkins, 1967
- Vogel S. *Life in moving fluids*. Princeton, NJ: Princeton University Press, 1981
- Rand PW, Lacombe, Hunt HE, Austin WH. Viscosity of normal human blood under normothermic and hypothermic conditions. *J Appl Physiol* 1964;19:117-122
- Evans RL, Kirkwood RB, Opsahl DG. The dynamic viscosity of some human blood. *Biorheology* 1971;8:125-128
- Barbee JH. The effect of temperature on the relative viscosity of human blood. *Biorheology* 1973;10:1-5
- Huang CR, Siskovic N, Robertson RW, et al. Quantitative characterization of thixotropy of whole human blood. *Biorheology* 1975;12:279-282
- Thurston GB. Rheological parameters for the viscosity, viscoelasticity and thixotropy of blood. *Biorheology* 1979;16:149-162
- Roach MR, Burton AC. The reason for the shape of the distensibility curves of arteries. *Can J Biochem Physiol* 1957;35:681-690
- Roach MR. The structure and elastic properties of arterial junctions. *Connect Tissue Res* 1986;15:77-84
- Skalak R, Keller SR, Secomb TW. Mechanics of blood flow. *J Biomech Eng* 1981;103(2):102-115
- Histand MB, Miller CW. A comparison of velocity profiles measured in unexposed and exposed arteries. *Biomed Sci Instrum* 1972;9:121-123
- Keller HM, Merer WE, Anliker M, et al. Noninvasive measurement of velocity profiles and blood flow in the common carotid artery by pulsed Doppler ultrasound. *Stroke* 1976;7:370-377
- Fitzgerald DE, O'Shaughnessy AM, Keaveny VT. Pulsed Doppler: determination of blood velocity and volume flow in normal and diseased carotid arteries in man. *Cardiovasc Res* 1982;16:220-224
- Benetos A, Simon A, Levenson J, et al. Pulsed Doppler: an evaluation of diameter blood velocity and blood flow of the common carotid artery in patients with isolated unilateral stenosis of the internal carotid artery. *Stroke* 1985;16:969-972
- Rittgers SE, Fei DY, Kraft KA, et al. Velocity profiles in stenosed tube models using magnetic resonance imaging. *Monogr Atheroscler* 1990;15:43-53
- Kraft KA, Fatouros DY, Fei SE, et al. MR imaging of model fluid velocity profiles. *Magn Reson Imaging* 1989;7:69-77
- Fei DY, Kraft KA, Fatouros P. Model studies of nonsteady flow using magnetic resonance imaging. *J Biomech Eng* 1990;112:93-108
- Liesch DW. *Blood Flow in Large Arteries*. Monographs in Atherosclerosis. Basel: Karger, 1990
- Edelman RR, Mattle HP, Atkinson DJ, et al. MR angiography. *AJR Am J Roentgenol* 1990;154:937-946
- Pericone JR, Siebert JE, Potchen EJ, et al. Three-dimensional phase-contrast MR angiography in the head and neck: preliminary report. *AJNR Am J Neuroradiol* 1990;11:457-466
- Ross JS, Masaryk TJ, Modic MT. Intracranial aneurysms: evaluation by MR angiography. *AJNR Am J Neuroradiol* 1990;11:449-456
- Stehbens WE. Etiology of intracranial berry aneurysms. *J Neurosurg* 1989;70:823-831
- Walker J. The amateur scientist: serious fun with polyx, Silly Putty slime, and other non Newtonian fluids. *Sci Am* 1986;186-196
- Clayton BR, Massey BS. Flow visualization in water: a review of techniques. *J Sci Instrum* 1967;44:2-11
- Merzkirch W. *Flow visualization*. New York: Academic Press, 1974
- Vogel S, Fader N. Visualization of low speed flow using suspended plastic particles. *Nature* 1966;209:186-187
- Vogel S, LaBarbera M. Simple flow tanks for research and teaching. *Bioscience* 1978;28:638-643
- Liesch D. Beruhungsloses Messen von Stromungsgeschwindigkeiten mittels Laser-Doppler-Anemometer. *Heizung und Luftung* 1978;29:341-344,383-387
- Durst F, Melling A, Whitlaw JH. *Principles and practice of Doppler anemometry*. London: Academic Press, 1976
- Hussain AKMF. Coherent structures: reality and myth. *Phys Fluids* 1983;26:2816-2850
- Liesch DW. Flow in tubes and arteries: a comparison. *Biorheology* 1986;23:395-433
- Kerber CW, Heilman CB. Flow dynamics in the human carotid artery. *AJNR Am J Neuroradiol* 1992;13:173-180
- Nelson TR. Physiologic complexity, scaling, and opportunities for imaging. *Invest Radiol* 1990;25:1140-1148
- Lieber BB. The decomposition of apparent stresses in disturbed pulsatile flow in the presence of large scale organized structures. *J Biomech* 1990;1047-1060
- Poiseuille JLM. Recherches experimentales sur le mouvement des liquides dans les tubes de tres petits diametres. *Academie des Sciences, Comptes Rendes* 1840;11:961-967
- Reynolds O. An experimental investigation of the circumstances which determine whether the motion of water shall be direct or sinous, and the law of resistance in parallel channels. *Trans R Soc Lond* 1883;174:935-982
- Karino T, Kwong HHN, Goldsmith HL. Particle flow behavior in models of branching vessels. I: vortices in ninety degree T-junctions. *Biorheology* 1979;16:231-248
- Karino T, Goldsmith HL, Motimiyo M. Flow patterns in simple and complex geometries. *Ann NY Acad Sci* 1987;516:422-541
- Famir M, Roach MR. Blood flow downstream of a two dimensional bifurcation. *J Theor Biol* 1973;42:33-48